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DEPARTMENT OF CIVIL ENGINEERING
AND ENGINEERING MECHANICS

AN APPROXIMATION IN PROBLEMS OF WAVE PROPAGATION IN COMPLEX VISCOELASTIC MEDIA

by

W. R. Spillers

and

H. H. Bleich

Office of Naval Research
Project NR 064-417
Contract Nonr-266(34)
Technical Report No. 13
CU 2-62 ONR-266(34)-CE

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Introduction

Waves in a viscoelastic rod have been studied by Morrison and others [1] using Laplace transform techniques. The form of the results obtained is so complex that it is difficult to obtain quantitative information without extensive numerical calculations; nor is it convenient to use this form in a Duhamel type integral. For this reason Bleich and Sackman [2,3] have proposed an exponential interpolation as an approximation in viscoelastic wave propagation problems. The above papers were limited to the consideration of materials having only one typical time constant like the standard solid. It is the purpose of this note to extend the idea of this exponential interpolation to materials which have a more complex response, containing two or more time constants. As before, [2], the interpolation makes use of the known discontinuity at the wave front and the known long time value of the response; further, it is anticipated that in the case of a material whose elements in creep or relaxation display j typical time constants, the response $F(t)$ in a wave propagation problem will have the character of Fig. 1 and may be approximated by an expression of the form

$$F(t) \sim F(\infty) - A_1 e^{-t/t_1} - A_2 e^{-t/t_2} - \dots - A_j e^{-t/t_j} \qquad (1)$$

where t is the time after arrival of the first signal at the location considered.

If $j = 1$ the two parameters A_1 and t_1 can be determined from the values $F(0)$ and $\dot{F}(0)$ which are easily extracted from the Laplace transform solution by asymptotic methods, but this method is insufficient if $j \geq 2$. The present note approaches the latter case by again starting with the Laplace transform. The transform of the response is then expanded in a series*, the inversion of which gives a series consisting of j exponential terms, e^{-t/t_j}, each multiplied by a power series in t. Comparing the leading terms of this expansion with Eq. (1), the values of t_j and A_j are obtained. The details of these steps are illustrated with the following example.

Example

Consider the problem of the propagation of waves in a semi-infinite $(x \leq 0)$ rod of a five-parameter viscoelastic material, initially at rest, subjected to a unit stress impulse at $t = 0$, $x = 0$. For an elastic rod with Young's modulus E and mass density ρ the stress is known to be

$$\sigma = H[t-x(\rho/E)^{\frac{1}{2}}] \tag{2}$$

where $H(t)$ is the Heavyside step function. Using the elastic-viscoelastic analogy [5] the Laplace transform of the stress in a viscoelastic rod subjected to similar conditions is

$$\bar{\sigma} = s^{-1} \exp[-sx(\rho/\bar{E})^{\frac{1}{2}}] \tag{3}$$

* The validity of the expansion has been discussed in [4].

where s is the transform parameter and $\bar{E}$ is a rational function of the transform parameter. For the five-parameter model shown in Fig. 2

$$\bar{E} = \frac{s^2(G_1+G_2+G_3) + s[\tau_1^{-1}(G_2+G_3) + \tau_2^{-1}(G_1+G_3)] + \tau_1^{-1}\tau_2^{-1}G_3}{(s+\tau_1^{-1})(s+\tau_2^{-1})}$$

$$= e\,\frac{(s+a)(s+b)}{(s+c)(s+d)} \qquad (4)$$

where a, b, c, d, and e are known constants. The inversion to be performed is

$$\sigma = \mathcal{L}^{-1}\left[s^{-1}\exp\left\{-sx\left(\frac{\rho}{e}\,\frac{s+c}{s+a}\,\frac{s+d}{s+b}\right)^{\frac{1}{2}}\right\}\right]$$

Using the shift theorem

$$\bar{\sigma} = s^{-1}\exp[-sx(\rho/e)^{\frac{1}{2}}]\exp\left[-sx\left(\frac{\rho}{e}\,\frac{s+c}{s+a}\,\frac{s+d}{s+b}\right)^{\frac{1}{2}} + sx(\rho/e)^{\frac{1}{2}}\right] \qquad (5)$$

$$\sigma = H[t-x(\rho/e)^{\frac{1}{2}}]\,F\,[t-x(\rho/e)^{\frac{1}{2}}] \qquad (6)$$

where $\bar{F}$ is well behaved as $s \to \infty$. This is a condition for the term by term inversion which follows. Using the expansions

$$e^x = \sum_{n=0}^{\infty} x^n/n! \qquad |x| < \infty$$

$$(1+x)^{\frac{1}{2}} = \sum_{n=0}^{\infty}\frac{(-x)^n\Gamma(n-1/2)}{\Gamma(-1/2)\Gamma(n+1)} = 1 + x/2 - x^2/8 + x^3/16 - \ldots$$

$$|x| < 1$$

one finds

$$F(t) = \mathcal{L}^{-1}\left[s^{-1}\exp\left\{-sx\left(\frac{\rho}{e}\left[1 + \frac{s(c+d-a-b) + cd-ab}{(s+a)(s+b)}\right]\right)^{\frac{1}{2}} + sx(\rho/e)^{\frac{1}{2}}\right\}\right] \qquad (7)$$

$$= \mathcal{L}^{-1}\left[s^{-1} \sum_{n=0}^{\infty} \frac{1}{n!} \left[-sx\left(\frac{\rho}{e}\right)^{\frac{1}{2}} \left\{ \sum_{m=0}^{\infty} \left(-\frac{s(c+d-a-b) + cd-ab}{(s+a)(s+b)}\right)^m \frac{\Gamma(m-1/2)}{\Gamma(-1/2)\Gamma(m+1)} -1\right\}\right]^n\right] \tag{8}$$

Eq. 8 may now be inverted term by term. It should be noted that each term in this equation can be reduced to a sum of terms of the form of a constant multiplied by s^{-1}, $(s+a)^{-n}$, or $(s+b)^{-n}$ therefore the inverted series will have the form

$$F(t) = \theta(x) + e^{-at} \sum_{n=0}^{\infty} \phi_n(x)t^n + e^{-bt} \sum_{n=0}^{\infty} \psi_n(x)t^n \tag{9}$$

Eq. (9) suggests an interpolation of the form of Eq. (1) (with $j = 2$) for any given value of x, in which $F(\infty)$ is the value of the response as $t \to \infty$ and therefore from Eq. (7) $F(\infty) = 1$. The sum $F(\infty) - A_1 - A_2$ represents the discontinuity at the wave front and may also be obtained from Eq. (7) using the usual asymptotic techniques and the expansion of $(1+x)^{\frac{1}{2}}$

$$F(t) = \mathcal{L}^{-1}\left[s^{-1}\exp\left\{-sx(\rho/e)^{\frac{1}{2}}[1+ \frac{s(c+d-a-b)+cd-ab}{2(s+a)(s+b)} + \dots]+sx(\rho/e)^{\frac{1}{2}}\right\}\right]$$

$$\begin{aligned} F(0) &= \lim_{s \to \infty} s\,\bar{F}(s) = \exp[-x(\rho/e)^{\frac{1}{2}}(c+d-a-b)/2] \\ &= F(\infty) - A_1 - A_2 \end{aligned} \tag{10}$$

Eq. (1) may be re-written

$$F(t) \sim 1 + \frac{F(0)-1}{1+\alpha} [\alpha e^{-at} + e^{-bt}] \tag{11}$$

where $\alpha = A_1/A_2$ remains to be determined. The first term of Eq. (8), $n = 0$, is s^{-1}. For $n = 1$, there is the term

$$\tfrac{1}{2}\, x(\rho/e)^{\frac{1}{2}}\, \frac{s(c+d-a-b) + cd-ab}{(s+a)(s+b)} \tag{12}$$

plus terms of lower order in s as $s \to \infty$. For all other n, the terms of Eq. (8) are of lower order in s as $s \to \infty$ than Eq. (12). For these reasons Eq. (12) is used to determine the constant α. The inverse transform of Eq. (12) is

$$\tfrac{1}{2}x(\rho/e)^{\frac{1}{2}}\left[\frac{-a(c+d-a-b)+cd-ab}{(-a+b)}\, e^{-at} + \frac{-b(c+d-a-b)+cd-ab}{(-b+a)}\, e^{-bt}\right] \tag{13}$$

or

$$\alpha = -\,\frac{-a(c+d-a-b)+cd-ab}{-b(c+d-a-b)+cd-ab} \tag{14}$$

As an example, let $G_2 = G_3 = 1$, $G_1 = 100$, $\tau_1^{-1} = C = 1$, $\tau_2^{-1} = d = 100$. It follows from the definition, Eq. (4), that $a = 99.029$, $b = .0099$, and $\alpha = .961$. At the point $x(\rho/e)^{\frac{1}{2}}\tau_1^{-1} = 1$ on the rod, the interpolation gives

$$\sigma \sim H(t-1)[1-.318(.961\, e^{-99.029(t-1)} + e^{-0.0099(t-1)})]$$

It may be noted that quite frequently α may be very much larger than unity, or smaller than unity. In such a case only one of the time constants really governs the response. However, it does not seem easy to predict such a situation on physical grounds prior to the analysis.

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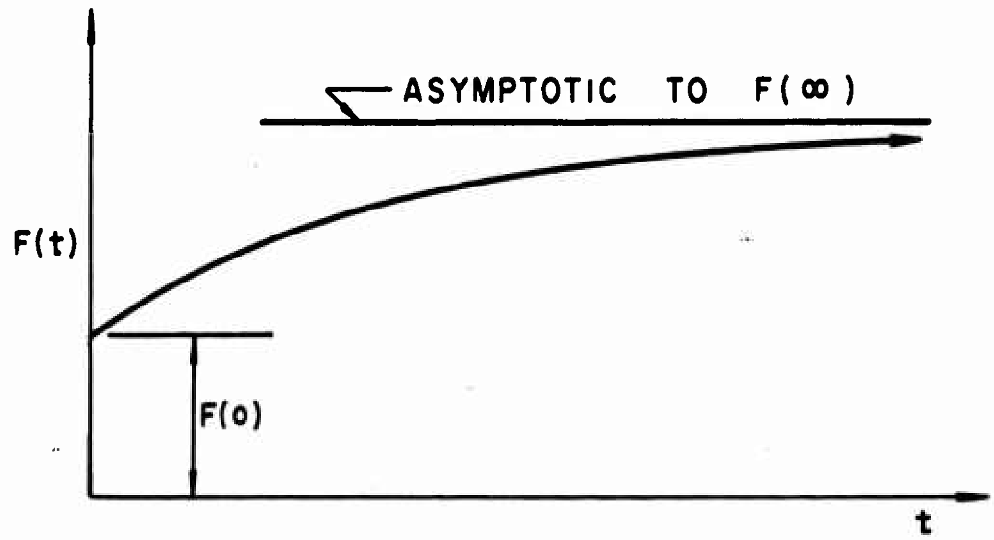

FIG. 1

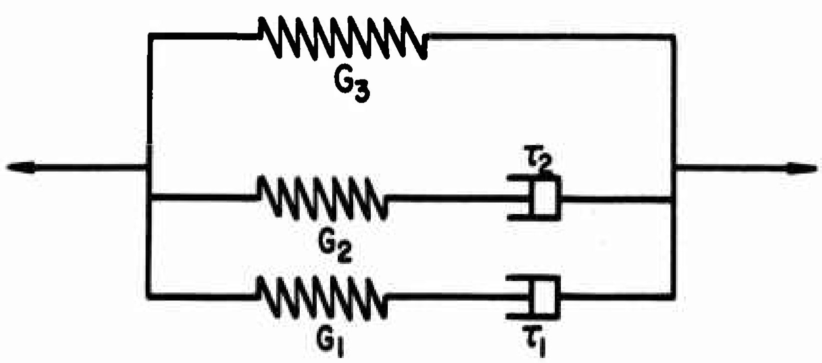

FIG. 2

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